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THE MEISSNER EFFECT IN A TWO FLAVOR LOFF COLOR SUPERCONDUCTOR

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Abstract

We calculate the magnetic polarization tensor of the photon and of the gluons in a two flavor color superconductor with a LOFF pairing that consists of a single plane wave. We show that at zero temperature and within the range of the values of the Fermi sea displacement that favors the LOFF state, all the eigenvalues of the magnetic polarization tensor are non-negative. Therefore the chromomagnetic instabilities pertaining to a gapless color superconductor disappear.

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1 Introduction

The LOFF (Larkin-Ovchinnikov-Fudde-Ferrell) state [1], [2] was introduced as a candidate superconducting state of an alloy containing paramagnetic impurities with a ferromagnetic spin alignment. The presence of a nonzero expectation value for the spins of the impurities and its corresponding coupling to the electrons gives rise to a displacement of the Fermi surfaces of the pairing electrons since they have opposite spins. The characteristic feature of this state is that the corresponding order parameter is not constant but depends on the coordinates. Though the experimental evidence for the LOFF state is still inconclusive, the subject has experienced a revived interest due to its relevance to the physics of color superconductors and compact stars [3].

The interaction between two quarks that renders quark matter at ultra high baryon density unstable to the formation of Cooper pairs and leads to the emergence of a color superconducting state, results from the perturbative one-gluon exchange in the color antisymmetric diquark channel. All quark flavors, u, d and s can be treated as massless since the Fermi energies of the quarks are much higher than Λ_{QCD}. In contrast at moderately high baryon densities, say the density inside a compact star, where the Fermi energies of the quarks are only few hundred MeV, instanton effects are expected to dominate the interaction responsible for the pairing of quarks [4], again in the color antisymmetric channel. Since the main pairing channel is between quarks of different flavors, factors such as the large mass of the s quark and the different magnitudes of electric charges of the u and d quarks together with the color/electric neutrality condition induce a considerable displacement between the Fermi momenta of the pairing quarks. This makes the search for a realistic color superconductivity phase challenging.

The LOFF state is one of several states that were proposed as a possible ground state for quark matter at moderately high baryon density [5], [6]. Others include the homogeneous gapless states (g2SC or gCFL) [7], [8], [9] and the heterogeneous mixture of BCS states without Fermi sea displacement and normal phases [12]. The gapless state is the analog of the Sarma state [10] for quark matter and a potential ground state since the Sarma instability can be removed by imposing the charge neutrality constraint [7], [11]. But a subsequent calculation revealed that the squares of the Meissner masses of the gluons were all negative signaling a chromomagnetic instability in the presence of the gauge field [13], [14], [15], [16]. This instability represents a major problem that needs to be resolved before we would be able to identify the ground state of the quark matter.

The mixed state on the other hand is free from chromomagnetic instabilities but it has the drawback that it is difficult to make a quantitative comparison of its free energy with one of the homogeneous CSC state.

In a previous paper [15], we explored the relationship between the chromomagnetic instability and the LOFF state. For a g2SC, we showed explicitly that the chromomagnetic instability associated with the 8th gluon implies the existence of a LOFF state with a lower free energy and a positive Meissner mass square for the 8th gluon excitation. But the possibility remains that the chromomagnetic instability will emerge when we calculate the Meissner masses of the gluons that are associated with the 4-7th generators of the color group. In this paper we shall continue this project by calculating the Meissner masses of all the gluons of a LOFF state. Our result shows that the chromomagnetic

instability is completely removed at zero temperature and in the LOFF window where the LOFF state is energetically favored for a given Fermi momentum displacement.

This paper is organized as follows: The free energy function of a two flavor LOFF CSC will be derived in section II. A discussion of its properties will also be included. In section III, the Meissner masses of all the gluonic excitations will be calculated. The analytic proof of the absence of the chromomagnetic instabilities for a small gap parameter Δ, at $T = 0$ will be presented in section IV and the numerical results for an arbitrary Δ will be discussed in section V, where we extend the chromagnetic instability free region to the entire LOFF window at $T = 0$. Finally, in section VI will summarize our results. Some technical details will be presented in the appendices. Unless explicitly stated, we shall work exclusively with the single plane wave ansatz for which analytical investigation can be carried out quite far. We shall also designate the symbol 2SC-LOFF for a two flavor LOFF color superconducting state.

2 The Free Energy Function of a 2SC-LOFF

We consider the pairing of two quark flavors, u and d. The NJL effective Lagrangian in the massless limit is given by [17],

$$\begin{aligned}\mathcal{L} = -\bar{\psi}\gamma_\mu\frac{\partial\psi}{\partial x_\mu} + \bar{\psi}\gamma_4\mu\psi \quad &+ \quad G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}\vec{\tau}\psi)^2] \\ &+ \quad G_D(\bar{\psi}_C\gamma_5\epsilon^c\tau_2\psi)(\bar{\psi}\gamma_5\epsilon^c\tau_2\psi_C)\end{aligned} \tag{1}$$

where ψ represents the quark fields and $\psi_C = C\tilde{\bar{\psi}}$ is its charge conjugate with $C = i\gamma_2\gamma_4$. All gamma matrices are hermitian, $(\epsilon^c)^{mn} = \epsilon^{cmn}$ is a 3×3 matrix acting on the red, green and blue color indices and the Pauli matrices $\vec{\tau}$ act on the u and d flavor (isospin) indices. The chemical potential is also a matrix in color-flavor space, i.e.

$$\mu = \bar{\mu} - \delta\tau_3 + \delta'\lambda_8 \tag{2}$$

with λ_l being the lth Gell-Mann matrix.

The most general expression of the order parameter of 2SC-LOFF is a superposition of plane waves of diquark condensate, i.e.

$$< \bar{\psi}(\vec{r})\gamma_5\lambda_2\tau_2\psi_C(\vec{r}) >= \sum_{\vec{q}} \Phi_{\vec{q}} e^{2i\vec{q}\cdot\vec{r}}, \tag{3}$$

In this article, we shall be mainly working with the type of the LOFF state that contains only one plane wave, more specifically the form of the order parameter will be given by

$$< \bar{\psi}(\vec{r})\gamma_5\lambda_2\tau_2\psi_C(\vec{r}) >= \Phi e^{2i\vec{q}\cdot\vec{r}}. \tag{4}$$

The condensate eq. (4) is invariant under a translation accompanied by a phase shift. Since the free energy depends on the magnitude of the condensate only, it is translationally invariant. But this is not clearly the case with the general form of the 2SC-LOFF condensate eq. (3).

Let's introduce a transformed quark field,

$$\chi(\vec{r}) = e^{-i\vec{q}\cdot\vec{r}}\psi(\vec{r}). \tag{5}$$

The NJL Lagrangian in terms of the new quark field reads

$$\begin{aligned}\mathcal{L} = -\bar{\chi}\gamma_\mu\frac{\partial\chi}{\partial x_\mu} - i\bar{\chi}\vec{\gamma}\cdot\vec{q}\chi + \bar{\chi}\gamma_4\mu\chi &+ G_S[(\bar{\chi}\chi)^2 + (\bar{\chi}\vec{\tau}\chi)^2] \\ &+ G_D(\bar{\chi}_C\gamma_5\epsilon^c\tau_2\chi)(\bar{\chi}\gamma_5\epsilon^c\tau_2\chi_C),\end{aligned} \tag{6}$$

while the condensate (4) takes the form

$$< \bar{\chi}(\vec{r})\gamma_5\lambda_2\tau_2\chi_C(\vec{r}) >= \Phi, \tag{7}$$

By ignoring the chiral condensate and expanding the NJL Lagrangian to linear order in the fluctuation

$$\bar{\psi}\gamma_5\lambda_2\tau_2\psi_C - \Phi, \tag{8}$$

we derive the following mean field expression for the NJL Lagrangian

$$\mathcal{L}_{MF} = -\frac{\Delta^2}{4G_D} - \bar{\chi}\gamma_\mu\frac{\partial\chi}{\partial x_\mu} - i\bar{\chi}\vec{\gamma}\cdot\vec{q}\chi + \bar{\chi}\gamma_4\mu\chi + \frac{1}{2}\Delta(-\bar{\chi}_C\gamma_5\lambda_2\tau_2\chi + \bar{\chi}\gamma_5\lambda_2\tau_2\chi_C), \tag{9}$$

where the gap parameter is given by $\Delta = 2G_D\Phi > 0$. The free energy $\mathcal{F}$ is given by the Euclidean path integral

$$\mathcal{F} = -k_BT\ln\Big[\int[d\psi d\bar{\psi}]\exp\Big(\int d^4x\mathcal{L}_{MF}\Big)\Big] \tag{10}$$

with $0 < x_4 < (k_BT)^{-1}$ and will be minimized at equilibrium. We find it convenient to work with the difference of the free energy density between the superconducting and normal phases in a grand canonical ensemble at the same δ and δ',

$$\Gamma = \frac{1}{\Omega}(\mathcal{F} - \mathcal{F}|_{\Delta=0}), \tag{11}$$

where Ω is the volume of the system. The equilibrium conditions read

$$\frac{\partial\Gamma}{\partial\Delta^2} = 0, \qquad \frac{\partial\Gamma}{\partial q} = 0, \tag{12}$$

and

$$\frac{\partial^2\Gamma}{(\partial\Delta^2)^2} > 0, \qquad \begin{vmatrix} \frac{\partial^2\Gamma}{(\partial\Delta^2)^2} & \frac{\partial^2\Gamma}{\partial\Delta^2\partial q} \\ \frac{\partial^2\Gamma}{\partial\Delta^2\partial q} & \frac{\partial^2\Gamma}{\partial q^2}\end{vmatrix} > 0. \tag{13}$$

It follows then that

$$\frac{\partial^2\Gamma}{\partial q^2} > 0 \tag{14}$$

at the minimum.

In terms of the Nambu-Gorkov representation of the quark field,

$$\Psi = \begin{pmatrix} \chi \\ \tau_2\chi_C \end{pmatrix}, \qquad \bar{\Psi} = (\bar{\chi}, \bar{\chi}_C\tau_2) \tag{15}$$

and its Fourier transform,

$$\Psi(\vec{r}) = \sqrt{k_BT}\sum_\nu \int \frac{d^3\vec{p}}{(2\pi)^3} e^{iPx}\Psi_P \tag{16}$$

we find that

$$\int d^4x\mathcal{L}_{MF} = \Omega\Big[-\frac{\Delta^2}{4G_D} + \frac{1}{2}\sum_\nu \int \frac{d^3\vec{p}}{(2\pi)^3}\bar{\Psi}_P\mathcal{S}^{-1}(P)\Psi_P\Big] \tag{17}$$

where $\mathcal{S}^{-1}(P)$ is the momentum representation of the inverse quark propagator

$$\begin{aligned}\mathcal{S}^{-1}(P) &= \begin{pmatrix} \not{P} - i\vec{\gamma}\cdot\vec{q} + \bar{\mu}\gamma_4 - \delta\gamma_4\tau_3 + \delta'\gamma_4\lambda_8 & \Delta\gamma_5\lambda_2 \\ -\Delta\gamma_5\lambda_2 & \not{P} + i\vec{\gamma}\cdot\vec{q} - \bar{\mu}\gamma_4 - \delta\gamma_4\tau_3 - \delta'\gamma_4\lambda_8 \end{pmatrix} \\ &= \not{P} - \delta\gamma_4\tau_3 + (-i\vec{\gamma}\cdot\vec{q} + \bar{\mu}\gamma_4 + \delta'\gamma_4\lambda_8)\rho_3 + i\Delta\gamma_5\lambda_2\rho_2. \end{aligned} \tag{18}$$

$\not{P} = -i\gamma_\mu P_\mu$, $P = (\vec{p}, -\nu)$ with $\nu = (2n+1)\pi k_BT$ being the Matsubara frequency, and we have introduced the Pauli matrices with respect to NG blocks, i.e.

$$\rho_1 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad \rho_2 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix} \qquad \rho_3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \tag{19}$$

The product of Dirac, color, flavor and NG matrices is a direct product and the total dimensionality of $\mathcal{S}^{-1}(P)$ is 4(Dirac) $\times$ 3(color) $\times$ 2(flavor) $\times$ 2(NG) = 48.

Carrying out the path integration in Eq. (10), we find

$$\Gamma = \frac{\Delta^2}{4G_D} - \frac{k_BT}{2}\sum_\nu \int \frac{d^3\vec{p}}{(2\pi)^3}\ln\frac{\det\mathcal{S}^{-1}(P)}{\det S^{-1}(P)}, \tag{20}$$

where $S(P) \equiv \mathcal{S}(P)|_{\Delta=0}$ is the free quark propagator.

The typical energy scale where the pairing occurs is characterized by the zero temperature gap parameter Δ_0 in the absence of a displacement, i.e. $\delta = \delta' = 0$. At weak coupling, $\Delta_0 << \bar{\mu}$ and the pairing interaction extends over a width of ω_0 on both sides of the Fermi level with $\Delta_0 << \omega_0 << \bar{\mu}$. Furthermore the displacement δ' that is induced by the condensate, is of higher order and can be neglected. As a result we have the only displacement parameter $\delta \sim \Delta_0$. Under this approximation, the quark propagator takes the approximate form

$$\mathcal{S}(P) \simeq \frac{(-i\vec{\gamma}\cdot\hat{p} + \rho_3\gamma_4)}{2}\frac{[|\vec{p} - \rho_3\vec{q}| - \bar{\mu} + \rho_3(i\nu - \delta\tau_3) + \Delta\lambda_2\gamma_5\gamma_4]}{(p-\bar{\mu})^2 + (i\nu - \delta\tau_3 - \hat{p}\cdot\vec{q})^2 + \Delta^2} \tag{21}$$

while the determinant in eq. (20) is given by

$$\det\mathcal{S}^{-1}(P) = (2\bar{\mu})^{24}\prod_{s,\lambda}[(p-\bar{\mu})^2 - (i\nu + s\delta - \hat{p}\cdot\vec{q})^2 + (\lambda\Delta)^2]^2 \tag{22}$$

where $s = \pm 1$ and $\lambda = \pm 1, 0$ refer to the eigenvalues of τ_3 and λ_2 respectively. A more detailed derivation of eqs. (21) and (22) can be found in appendix A. It follows from the analytic continuation of (21) to the real energy, $i\nu \to p_0$, that the low-lying excitation spectrum reads

$$E^{\pm}_{\vec{p}} = \sqrt{(p-\bar{\mu})^2 + \Delta^2} \pm \delta - \hat{p}\cdot\vec{q} \tag{23}$$

for red and green colors and $E_{\vec{p}} = |p-\bar{\mu}| \pm \delta - \hat{p}\cdot\vec{q}$ for blue color. The excitations (23) are fully gapped if $\Delta > \delta + q$. $E^{-}_{\vec{p}}$ becomes gapless when $|\delta - q| < \Delta < \delta + q$ and both $E^{\pm}_{\vec{p}}$ become gapless when $q > \delta$ and $q - \delta < \Delta < q + \delta$ [2].

Substituting (22) into (20) we obtain that

$$\Gamma = \frac{\Delta^2}{4G_D} - \frac{\bar{\mu}^2 k_B T}{\pi^2} \sum_{|\nu|<\omega_0} \int_{-1}^{1} d\cos\theta \int_{-\infty}^{\infty} d\xi \ln \frac{\xi^2 - z^2(\nu,\theta,\Delta)}{\xi^2 - z^2(\nu,\theta,0)} \tag{24}$$

with $\xi = p - \bar{\mu}$ and $z(\nu,\theta,\Delta) = \sqrt{(i\nu + \delta - q\cos\theta)^2 - \Delta^2}$. The contributions from the eigenvalues of τ_3 with opposite sign are equal if we reverse the signs of the Matsubara frequency and simultaneously perform the transformation $\theta \to \pi - \theta$. This will also be the case with the calculation of the Meissner masses in the next section.

We shall employ the phase conventions that are determined by

$$\mathrm{Im} z(\nu,\theta,\Delta) > 0 \text{ for } \nu > 0 \ , \tag{25}$$

with $\zeta(-\nu,\theta,\Delta) = \zeta^{\star}(\nu,\theta,\Delta)$. Let's now proceed and perform the ξ integral by closing the contour around one of the branch cuts of the logarithm. As a result we obtain the expression

$$\Gamma = \frac{\Delta^2}{4G_D} - \frac{4\bar{\mu}^2 k_B T}{\pi} \mathrm{Im} \int_{-1}^{1} d\cos\theta \sum_{0<\nu<\omega_0} [z(\nu,\theta,\Delta) - z(\nu,\theta,0)]. \tag{26}$$

At zero temperature, $T = 0$, we have

$$k_B T \sum_{0<\nu<\omega_0} (...) = \int_{0^+}^{\omega_0} \frac{d\nu}{2\pi} (...) \tag{27}$$

and

$$\begin{aligned}\Gamma &= \frac{\Delta^2}{4G_D} - \frac{2\bar{\mu}^2}{\pi^2}\Delta^2\Big(\ln\frac{2\omega_0}{\Delta} + \frac{1}{2}\Big) \\ &+ \frac{\bar{\mu}^2}{\pi^2}\int_{-1}^{1} d\cos\theta \Big[\theta(|\delta_\theta| - \Delta)(\Delta^2 \ln\frac{|\delta_\theta| + \sqrt{\delta_\theta^2 - \Delta^2}}{\Delta} - |\delta_\theta|\sqrt{\delta_\theta^2 - \Delta^2}) + \delta_\theta^2\Big]\end{aligned} \tag{28}$$

where $\delta_\theta = (1 - \rho\cos\theta)\delta$ with $\rho = q/\delta$, being the normalized diquark momentum. The limit of the angular integral depends on the gapless region of the excitation spectrum (23) and will be discussed in detail in the appendix B. In terms of the gap parameter Δ_0 at $\delta = 0$, given by the standard BCS gap equation

$$\frac{1}{4G_D} = \frac{2\bar{\mu}^2}{\pi^2} \ln \frac{2\omega_0}{\Delta_0}, \tag{29}$$

we have

$$\Gamma = \frac{2\bar{\mu}^2}{\pi^2}\{\Delta^2\Big(\ln\frac{\Delta}{\Delta_0} - \frac{1}{2}\Big) + \frac{(\rho+1)^3}{4\rho}\delta^2\Big[(1-x_1^2)\ln\frac{1+x_1}{1-x_1} + \frac{2}{3}(2x_1^3 - 3x_1 + 1)\Big]$$
$$+ \frac{(\rho-1)^3}{4\rho}\delta^2\Big[(1-x_2^2)\ln\frac{1+x_2}{1-x_2} + \frac{2}{3}(2x_2^3 - 3x_2 + 1)\Big]\}, \quad (30)$$

where x_1 and x_2 are the same dimensionless parameters introduced in Ref. [2], i.e.

$$x_1 = \theta\Big(1 - \frac{\Delta}{(\rho+1)\delta}\Big)\sqrt{1 - \frac{\Delta^2}{(\rho+1)^2\delta^2}} \quad (31)$$

and

$$x_2 = \theta\Big(1 - \frac{\Delta}{|\rho-1|\delta}\Big)\sqrt{1 - \frac{\Delta^2}{(\rho-1)^2\delta^2}}. \quad (32)$$

Differentiating eq. (30) with respect to Δ^2 and ρ, we derive the equilibrium equations

$$\frac{\partial\Gamma}{\partial\Delta^2} = \frac{2\bar{\mu}^2}{\pi^2}\Big[\ln\frac{\Delta}{\Delta_0} - \frac{\rho+1}{4\rho}\Big(\ln\frac{1-x_1}{1+x_1} + 2x_1\Big) - \frac{\rho-1}{4\rho}\Big(\ln\frac{1-x_2}{1+x_2} + 2x_2\Big)\Big] = 0 \quad (33)$$

and

$$\frac{\partial\Gamma}{\partial\rho} = \frac{4\bar{\mu}^2\delta^2}{3\pi^2}\rho\{1 - \frac{(\rho+1)^2}{8\rho^3}\Big[3(1-x_1^2)\ln\frac{1+x_1}{1-x_1} + 4(\rho+1)x_1^3 - 6x_1\Big]$$
$$- \frac{(\rho-1)^2}{8\rho^3}\Big[-3(1-x_2^2)\ln\frac{1+x_2}{1-x_2} + 4(\rho-1)x_2^3 + 6x_2\Big]\} = 0. \quad (34)$$

If we subsitute into (30) the solution to the gap equation (33), we find

$$\Gamma = \frac{2\bar{\mu}^2}{\pi^2}\{\delta^2 + \frac{1}{3}\rho^2\delta^2 - \frac{1}{2}\Delta^2 - \frac{1}{6\rho}[(\rho+1)^3x_1^3 + (\rho-1)^3x_2^3]\delta^2\}. \quad (35)$$

Furthermore, the second order derivatives provide us with the expressions

$$\frac{\partial^2\Gamma}{(\partial\Delta^2)^2} = \frac{\bar{\mu}^2}{2\pi^2\rho\delta^2}\Big[\frac{1}{(\rho+1)(1+x_1)} + \frac{1}{(\rho-1)(1+x_2)}\Big], \quad (36)$$

$$\frac{\partial^2\Gamma}{\partial\rho^2} = \frac{4\bar{\mu}^2\delta^2}{3\pi^2}\Big[1 + \frac{3(\rho+1)^2}{4\rho^3}(1-x_1^2)\ln\frac{1+x_1}{1-x_1} - \frac{3(\rho-1)^2}{4\rho^3}(1-x_2^2)\ln\frac{1+x_2}{1-x_2}$$
$$+ \frac{(\rho+1)^3}{\rho^3}x_1^3 - \frac{3(\rho+1)(\rho^2+\rho+1)}{2\rho^3}x_1 + \frac{(\rho-1)^3}{\rho^3}x_2^3 - \frac{3(\rho-1)(\rho^2-\rho+1)}{2\rho^3}x_2\Big] \quad (37)$$

and

$$\frac{\partial^2\Gamma}{\partial\Delta^2\partial\rho} = \frac{\bar{\mu}^2}{2\pi^2\rho^2}\Big[\ln\frac{1-x_1}{1+x_1} - \ln\frac{1-x_2}{1+x_2} + 2(\rho+1)x_1 + 2(\rho-1)x_2\Big]. \quad (38)$$

Our expression of the free energy function eq. (35) coincides with the expression derived in [2] if we perform the following modifications: replace the Fermi velocity in their formula with one and multiply it by a factor of four. This factor accounts for the four pairing configurations, $(r, u_R) - (g, d_R)$, $(r, u_L) - (g, d_L)$, $(r, d_R) - (g, u_R)$ and $(r, d_L) - (g, u_L)$, per momentum for quarks versus one pairing configuration, spin up-spin down, per momentum for electrons, where the subscripts R and L refer to the helicities of the pairing quarks.

3 The Meissner Masses

In this section we shall consider the coupling of the quark fields to external gauge fields. The mean field NJL Lagrangian is modified

$$\begin{aligned}\mathcal{L} &= -\frac{\Delta^2}{4G_D} + \bar{\chi}\gamma_\mu\Big(\frac{\partial}{\partial x_\mu} - igA^l_\mu T_l - ieQA_\mu\Big)\chi - \bar{\chi}\vec{\gamma}\vec{q}\chi + \bar{\chi}\gamma_4\mu\chi \\ &+ \Delta(-\bar{\chi}_C\gamma_5\lambda_2\tau_2\chi + \bar{\chi}\gamma_5\lambda_2\tau_2\chi_C). \end{aligned} \tag{39}$$

A^l_μ and A_μ represent the gluon and photon gauge potentials respectively. The Lagrangian is invariant under $SU(3)_c \times U(1)_{em}$ transformations. The $SU(3)_c$ generators and the $U(1)_{em}$ charge operator can be written as

$$T_l = \frac{1}{2}\lambda_l, \qquad Q = \frac{1}{6} + \frac{1}{2}\tau_3. \tag{40}$$

The squared Meissner masses are the eigenvalues of the Meissner tensors, defined by

$$(M^2)^{l'l}_{ij} = \lim_{K\to 0}\Pi^{l'l}_{ij}(K), \tag{41}$$

where $\Pi^{l'l}_{ij}(K)$ is the gluon/photon magnetic polarization tensor given by

$$\begin{aligned}\Pi^{l'l}_{ij}(K) &= \frac{1}{2}\int_0^{(k_BT)^{-1}} dx_4 \int d^3\vec{r}e^{-iK\cdot x} < J^{l'}_i(\vec{r},\tau)J^l_j(0,0) >_C \\ &= -\frac{k_BT}{2}\sum_\nu\int\frac{d^3\vec{p}}{(2\pi)^3}Tr\Gamma^{l'}_i\mathcal{S}(P)\Gamma^l_j\mathcal{S}(P+K), \end{aligned} \tag{42}$$

where $< ... >_C$ stands for the connected Green's functions, i.e. $< AB >_C = < AB > - < A >< B >$ and $K = (k\hat{z}, -\omega)$ is the Euclidean four momentum either of the gluon or of the photon. The current operator reads $\vec{J}^l = \frac{1}{2}\bar{\Psi}\vec{\Gamma}^l\Psi$, where the quark-gluon/photon vertex with respect to the NG basis (15) takes the form

$$\Gamma^l_\mu = \begin{pmatrix} \gamma_\mu\mathcal{T}_l & 0 \\ 0 & -\gamma_\mu\tau_2\tilde{\mathcal{T}}_l\tau_2 \end{pmatrix}, \tag{43}$$

where $l = 1, 2, ..., 9$ and the tilde denotes transpose. Here we have introduced a new set of generators, $\mathcal{T}_l$, that mixes the gluons with photon, in accordance with the residual symmetry in the presence of the condensate (4)[18], i.e.

$$\mathcal{T}_l = T_l \tag{44}$$

for $l = 1, ..., 7$,

$$\mathcal{T}_8 = T_8\cos\theta + \frac{e}{g}Q\sin\theta \tag{45}$$

and

$$\mathcal{T}_9 = -\frac{g}{e}T_8\sin\theta + Q\cos\theta \tag{46}$$

with $\cos\theta = \frac{\sqrt{3}g}{\sqrt{3g^2+e^2}}$ and $\sin\theta = \frac{e}{\sqrt{3g^2+e^2}}$. The condensate is invariant under the residual gauge transformations $SU(2)\times\mathcal{U}(1)$, where $SU(2)$ is generated by $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{T}_3$ and $\mathcal{U}(1)$ by $\mathcal{T}_9$. We shall refer to the photon-gluon mixture associated to $\mathcal{T}_8$ as the 8th gluon.

Each component of the Meissner tensor is a matrix with respect to the generators of the gauge group, eqs.(44), (45) and (46). A simplification follows from the observation that the isospin part of the quark-photon vertex is the unit matrix with respect to the NG blocks ($-\tau_2\tilde{\tau}_3\tau_2 = \tau_3$) and consequently it contributes to the integrand of the Meissner tensor a total derivative, i.e.

$$\int \frac{d^3\vec{p}}{(2\pi)^3} Tr\gamma_j\tau_3\mathcal{S}(P)\mathcal{M}\mathcal{S}(P) = i\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{\partial}{\partial p_j} Tr\tau_3\mathcal{M}\mathcal{S}(P), \tag{47}$$

where $\mathcal{M}$ is an arbitrary matrix. Therefore the isospin term of the charge operator (40) can be omitted and we have effectively $Q = \frac{1}{6}$ [15, 16].

Next we shall prove that the Meissner tensor is diagonal with respect to the indices l and l', more specifically that it has the following form

$$(M^2)^{11}_{ij} = (M^2)^{22}_{ij} = (M^2)^{33}_{ij} = (M^2)^{99}_{ij} = 0, \tag{48}$$

$$(M^2)^{44}_{ij} = (M^2)^{55}_{ij} = (M^2)^{66}_{ij} = (M^2)^{77}_{ij} \equiv (m^2)_{ij}, \tag{49}$$

and

$$(M^2)^{88}_{ij} \equiv (m'^2)_{ij}. \tag{50}$$

The relations above follow in a straightforward manner from symmetry arguments, in particular the residual $SU(2)$ symmetry. We proceed to divide the set of the nine generators into three subsets. Subset I includes the unbroken generators $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$, subset II includes the broken generators $\mathcal{T}_4$, $\mathcal{T}_5$, $\mathcal{T}_6$ and $\mathcal{T}_7$ while subset III includes the broken $\mathcal{T}_8$ and the unbroken $\mathcal{T}_9$ generators. Under an $SU(2)$ transformation, $\mathcal{T}_l \to u\mathcal{T}_l u^\dagger$, the generators in the subset I transform as the adjoint representation, the generators in the subset II as the fundamental one while the generators in the subset III remain invariant. Therefore the indices l' and l which correspond to generators from different subsets do not mix in the expression for $(M^2)^{l'l}_{ij}$. The mass matrix vanishes within the subset I because of the unbroken gauge symmetry. As for generators of the subset II, we proceed to relabel its members according to $\mathcal{J}_1 = T_4$, $\mathcal{J}_{\bar{1}} = T_5$, $\mathcal{J}_2 = T_6$ and $\mathcal{J}_{\bar{2}} = T_7$. Introducing

$$\phi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{51}$$

we have

$$\mathcal{J}_\alpha = \frac{1}{2}\begin{pmatrix} 0 & \phi_\alpha \\ \phi^\dagger_\alpha & 0 \end{pmatrix}, \tag{52}$$

and similar expressions for $\mathcal{J}_{\bar{\alpha}}$ with $\phi_{\bar{\alpha}} = -i\phi_\alpha$, where the block decomposition is with respect to color indices. Since $\mathcal{J}_\alpha$ is symmetric and $\mathcal{J}_{\bar{\alpha}}$ is antisymmetric, there is no mixing between $\mathcal{J}_\alpha$ and $\mathcal{J}_{\bar{\beta}}$, following from the properties of the matrix $C = i\gamma_2\gamma_4$, i.e. $C\tilde{\gamma}_\mu C^{-1} = -\gamma_\mu$ and

$$\rho_3 C\tilde{\mathcal{S}}(P)(\rho_3 C)^{-1} = -\mathcal{S}(P), \tag{53}$$

a consequence of the symmetry under a combined transformation of a space inversion, a time reversal operation and a γ_5 multiplication (see appendix C for details). The identity of the matrix elements $(M^2)^{\alpha\beta}$ and $(M^2)^{\bar{\alpha}\bar{\beta}}$ follows from the equivalence of the doublet representation supported by $\mathcal{T}_\alpha$ and the anti-doublet one by $\mathcal{T}_{\bar{\alpha}}$. Finally in the subset III, the color matrices never mix the red and green components with the blue ones. The latter do not carry a condensate. It follows then that we need only to work within the red-green subspace, in which $\mathcal{T}_9$ does not contribute and $\mathcal{T}_8$ may be replaced by a factor $\frac{1}{6g}\sqrt{3g^2+e^2}$.

The above argument on the group theoretic structure of the Meissner tensor leads also to its relation with the momentum susceptibility introduced in Ref. [15],

$$(m')^2_{ij} = \frac{1}{12}\Big(g^2+\frac{e^2}{3}\Big)\frac{\partial^2\Gamma}{\partial q_i\partial q_j}. \tag{54}$$

Because of the single plane wave ansatz, eq. (4), each of the nontrivial tensors $(m^2)_{ij}$ and $(m'^2)_{ij}$ can further be decomposed into its transverse and logitudinal components, i.e.

$$(m^2)_{ij} = A(\delta_{ij}-\hat{q}_i\hat{q}_j)+B\hat{q}_i\hat{q}_j \tag{55}$$

$$(m'^2)_{ij} = C(\delta_{ij}-\hat{q}_i\hat{q}_j)+D\hat{q}_i\hat{q}_j. \tag{56}$$

In particular, eq. (54) implies that

$$C = \frac{g^2}{12\rho\delta^2}\Big(1+\frac{e^2}{3g^2}\Big)\frac{\partial\Gamma}{\partial\rho} = \frac{g^2}{6\delta^2}\Big(1+\frac{e^2}{3g^2}\Big)\frac{\partial\Gamma}{\partial\rho^2} \tag{57}$$

and

$$D = \frac{g^2}{12\delta^2}\Big(1+\frac{e^2}{3g^2}\Big)\frac{\partial^2\Gamma}{\partial\rho^2}. \tag{58}$$

It follows from eqs. (57) and (34) at $\rho\neq 0$ that the Meissner tensor of the 8th gluon is longitudinal at the LOFF equilibrium. So is, the Meissner tensor of an electronic LOFF superconductor. This conclusion of ours disagrees with that of [2], where a nonzero transverse Meissner mass was reported. The above property does not contradict with the δ_{ij} structure of the Meissner tensor of a homogeneous superconductor since it corresponds to the trivial solution, $\rho=0$ of eq. (34), where $\frac{\partial\Gamma}{\partial\rho^2}\neq 0$.

In order to calculate the quantities A, B, C and D we follow the same procedure that we used to calculate the free energy in the previous section. In order to improve the UV convergence, we employ the technique of [19] and subtract from the polarization tensor of the superconducting phase, (42), the correponding expression of the normal phase with the same $\bar{\mu}$ and δ, which is zero at $K=0$, and write

$$\Pi^{l'l}_{ij}(0) = -\frac{k_BT}{2}\sum_\nu\int\frac{d^3\vec{p}}{(2\pi)^3}Tr[\Gamma^{l'}_i\mathcal{S}(P)\Gamma^l_j\mathcal{S}(P)-\Gamma^{l'}_iS(P)\Gamma^l_jS(P)], \tag{59}$$

where $S(P)=\mathcal{S}(P)\mid_{\Delta=0}$. Then the weak coupling approximation of the propagator, eq.(21), is inserted. We take the z-axis to be parallel to $\vec{q}$ and carry out the integration over the azimuthal angle and the magnitude of $\vec{p}$. We are then left with the integration

over the meridian angle and the summation over the Matsubara energies. Finally, we project out the transverse and longitudinal components and obtain the expressions

$$A = \frac{3}{4}\int_{-1}^{1} d\cos\theta \sin^2\theta F(\theta,\Delta), \tag{60}$$

$$B = \frac{3}{2}\int_{-1}^{1} d\cos\theta \cos^2\theta F(\theta,\Delta), \tag{61}$$

$$C = \frac{3}{4}\int_{-1}^{1} d\cos\theta \sin^2\theta G(\theta,\Delta), \tag{62}$$

and

$$D = \frac{3}{2}\int_{-1}^{1} d\cos\theta \cos^2\theta G(\theta,\Delta), \tag{63}$$

where

$$F(\theta,\Delta) = \frac{2g^2\bar{\mu}^2 k_B T}{3\pi}\mathrm{Im}\sum_{\nu>0}\frac{1}{z(\nu,\theta,\Delta)}\frac{i\nu+\delta_\theta - z(\nu,\theta,\Delta)}{i\nu+\delta_\theta + z(\nu,\theta,\Delta)} \tag{64}$$

and

$$G(\theta,\Delta) = \frac{2g^2\bar{\mu}^2\Delta^2 k_B T}{9\pi}\Big(1+\frac{e^2}{3g^2}\Big)\mathrm{Im}\sum_{\nu>0}\frac{1}{z^3(\nu,\theta,\Delta)}. \tag{65}$$

At $T=0$, the summation over ν becomes an integral and we find

$$F(\theta,\Delta) = \frac{g^2\bar{\mu}^2}{3\pi^2}\Big[\frac{1}{2} - \frac{\delta_\theta^2}{\Delta^2} + \theta(|\delta_\theta|-\Delta)\frac{\delta_\theta^2}{\Delta^2}\sqrt{1-\frac{\Delta^2}{\delta_\theta^2}}\Big] \tag{66}$$

and

$$G(\theta,\Delta) = \frac{g^2\bar{\mu}^2}{9\pi^2}\Big(1+\frac{e^2}{3g^2}\Big)\Big[1-\frac{\theta(|\delta_\theta|-\Delta)}{\sqrt{1-\frac{\Delta^2}{\delta_\theta^2}}}\Big]. \tag{67}$$

We recognize that eq. (66) and (67) are precisely the expressions that Huang and Shovkovy obtained for the Meissner masses [13] when their parameter $\delta\mu$ is replaced by an angle-dependent one, $|\delta_\theta|$. By carrying out the angular integration as it is described in detail in the appendix B, we find

$$\begin{aligned} A &= \frac{g^2\bar{\mu}^2}{6\pi^2}\Big[1-\frac{2\delta^2}{\Delta^2}\Big(1+\frac{1}{5}\rho^2\Big) - \frac{3\Delta^2}{16\rho^3\delta^2}\Big(\ln\frac{1+x_1}{1-x_1} - \ln\frac{1+x_2}{1-x_2}\Big) \\ &+ \frac{\delta^2}{5\rho^3\Delta^2}(\rho+1)^5x_1^5 - \frac{\delta^2}{8\rho^3\Delta^2}(\rho+1)^4x_1(5x_1^2-3) \\ &+ \frac{\delta^2}{5\rho^3\Delta^2}(\rho-1)^5x_2^5 + \frac{\delta^2}{8\rho^3\Delta^2}(\rho-1)^4x_2(5x_2^2-3)\Big], \end{aligned} \tag{68}$$

$$\begin{aligned} B &= \frac{g^2\bar{\mu}^2}{6\pi^2}\Big[1-\frac{2\delta^2}{\Delta^2}\Big(1+\frac{3}{5}\rho^2\Big) + \frac{3\Delta^2}{8\rho^3\delta^2}\Big(\ln\frac{1+x_1}{1-x_1} - \ln\frac{1+x_2}{1-x_2}\Big) \\ &+ \frac{\delta^2}{\rho\Delta^2}(\rho+1)^3x_1^3 - \frac{2\delta^2}{5\rho^3\Delta^2}(\rho+1)^5x_1^5 + \frac{\delta^2}{4\rho^3\Delta^2}(\rho+1)^4x_1(5x_1^2-3) \\ &+ \frac{\delta^2}{\rho\Delta^2}(\rho-1)^3x_2^3 - \frac{2\delta^2}{5\rho^3\Delta^2}(\rho-1)^5x_2^5 - \frac{\delta^2}{4\rho^3\Delta^2}(\rho-1)^4x_2(5x_2^2-3)\Big], \end{aligned} \tag{69}$$

$$C = \frac{g^2\bar{\mu}^2}{9\pi^2}\Big(1+\frac{e^2}{3g^2}\Big)\{1 - \frac{(\rho+1)^2}{8\rho^3}\Big[3(1-x_1^2)\ln\frac{1+x_1}{1-x_1}+4(\rho+1)x_1^3-6x_1\Big]$$
$$- \frac{(\rho-1)^2}{8\rho^3}\Big[-3(1-x_2^2)\ln\frac{1+x_2}{1-x_2}+4(\rho-1)x_2^3+6x_2\Big]\} \quad (70)$$

and

$$D = \frac{g^2\bar{\mu}^2}{9\pi^2}\Big(1+\frac{e^2}{3g^2}\Big)\{1 + \frac{\rho+1}{4\rho^3}\Big[3(\rho+1)(1-x_1^2)\ln\frac{1+x_1}{1-x_1}$$
$$+ 4(\rho+1)^2x_1^3-6(\rho^2+\rho+1)x_1\Big]$$
$$+ \frac{\rho-1}{4\rho^3}\Big[-3(\rho-1)(1-x_2^2)\ln\frac{1+x_2}{1-x_2}$$
$$+ 4(\rho-1)^2x_2^3-6(\rho^2-\rho+1)x_2\Big]\}. \quad (71)$$

Comparing (70) and (71) with (34) and (37), we confirm explicitly the relation between the Meissner tensor of the 8th gluon and the momentum susceptibility, eq. (54).

4 The Meissner Effect for a Small Gap Parameter

In the last two sections we derived the expressions for the free energy and the Meissner masses of the gluons for a 2SC-LOFF of arbitrary Δ, δ and ρ. In this section we shall proceed to address the issue of the chromomagnetic instability at equilibrium. Initially we shall consider the case $\Delta << \delta$, -the gap parameter much smaller than the displacement. This is the situation near a second order phase transition to the normal phase. For a LOFF state consisting of a single plane wave at $T=0$ this occurs at [2]

$$\delta = \delta_c \simeq 0.754\Delta_0, \quad (72)$$

which is also the maximum for the LOFF condensate, eq. (4). Although it has been argued that the single plane wave ansatz is not energetically favorable compared with the multi plane wave ansatz in this neighbourhood and that the transition to the normal phase may be a first order one [21], we shall still pursue the small gap expansion since it provides a full analytic treatment and consequently can be very instructive. We shall focus on the LOFF state at $T=0$.

Let's expand the free energy, eq. (30) to quartic power of the small parameter, $\hat{\Delta} \equiv \Delta/\delta$,

$$\Gamma = \frac{2\bar{\mu}^2\delta^2}{\pi^2}\Big[\alpha\hat{\Delta}^2+\frac{1}{2}\beta\hat{\Delta}^4\Big] \quad (73)$$

with,

$$\alpha = \ln\frac{2\delta}{\Delta_0}-1+\frac{\rho+1}{2\rho}\ln(\rho+1)+\frac{\rho-1}{2\rho}\ln|\rho-1| \quad (74)$$

and

$$\beta = \frac{1}{4(\rho^2-1)}. \quad (75)$$

The corresponding expansions of the quantities A and B read

$$A = \frac{g^2\bar{\mu}^2}{6\pi^2}(a_0\hat{\Delta}^2 + a_1\hat{\Delta}^4 + ...), \tag{76}$$

$$B = \frac{g^2\bar{\mu}}{6\pi^2}(b_0\hat{\Delta}^2 + b_1\hat{\Delta}^4 + ...), \tag{77}$$

where

$$a_0 = \frac{3}{8\rho^3}\Big(2\rho - \ln\frac{\rho+1}{\rho-1}\Big), \tag{78}$$

$$a_1 = -\frac{1}{16\rho^3}\Big(\frac{1}{\rho-1} + \frac{1}{\rho+1}\Big) - \frac{1}{32\rho^3}\Big[\frac{1}{(\rho-1)^2} - \frac{1}{(\rho+1)^2}\Big], \tag{79}$$

$$b_0 = \frac{3}{4\rho^2}\Big(\frac{2-\rho^2}{\rho^2-1} + \frac{1}{\rho}\ln\frac{\rho+1}{\rho-1}\Big) \tag{80}$$

and

$$b_1 = -\frac{1}{8\rho^3}\Big(\frac{1}{\rho-1} + \frac{1}{\rho+1}\Big) - \frac{1}{16\rho^3}\Big[\frac{1}{(\rho-1)^2} + \frac{1}{(\rho+1)^2}\Big] + \frac{1}{16\rho}\Big[\frac{1}{(\rho+1)^3} + \frac{1}{(\rho-1)^3}\Big] \tag{81}$$

Similarly, we find

$$C = \frac{g^2\bar{\mu}^2}{9\pi^2}\Big(1 + \frac{e^2}{3g^2}\Big)(c_0\hat{\Delta}^2 + c_1\hat{\Delta}^4 + ...), \tag{82}$$

$$D = \frac{g^2\bar{\mu}^2}{9\pi^2}\Big(1 + \frac{e^2}{3g^2}\Big)(d_0\hat{\Delta}^2 + d_1\hat{\Delta}^4 + ...). \tag{83}$$

where

$$c_0 = \frac{3}{4\rho^3}\Big(2\rho - \ln\frac{\rho+1}{\rho-1}\Big) = 2a_0, \tag{84}$$

$$c_1 = \frac{3}{32\rho^3}\Big[-\frac{2}{\rho+1} - \frac{2}{\rho-1} + \frac{1}{(\rho+1)^2} - \frac{1}{(\rho-1)^2}\Big], \tag{85}$$

$$d_0 = \frac{3}{2\rho^2}\Big(\frac{2-\rho^2}{\rho^2-1} + \frac{1}{\rho}\ln\frac{\rho+1}{\rho-1}\Big) = 2b_0 \tag{86}$$

and

$$d_1 = \frac{3}{16\rho^3}\Big[\frac{3}{\rho+1} + \frac{3}{\rho-1} - \frac{3}{(\rho+1)^2} + \frac{3}{(\rho-1)^2} + \frac{1}{(\rho+1)^3} + \frac{1}{(\rho-1)^3}\Big]. \tag{87}$$

The validity of the relations $c_0 = 2a_0$ and $d_0 = 2b_0$ is not restricted to zero temperature and the weak coupling approximation employed here. If we expand the integrand of the Meissner mass tensor (59) to the order $\hat{\Delta}^2$, we find that the loop integrals for a_0 and b_0 are related to those for c_0 and d_0 through integration by parts [15]. The same argument also leads to identical criteria for the chromomagnetic instability of the 4-7th gluons and the 8th gluon near T_c for g2SC. Minimization of the free energy with respect to Δ and ρ yields

$$\hat{\Delta}^2 = 4(\rho_c^2 - 1)\Big(1 - \frac{\delta}{\delta_c}\Big), \tag{88}$$

and

$$\rho = \rho_c + \frac{1}{4}\frac{\rho_c}{\rho_c^2 - 1}\hat{\Delta}^2, \tag{89}$$

where $\rho_c \simeq 1.20$ is the solution of the equation [2]

$$\frac{1}{2\rho_c}\ln\frac{\rho_c + 1}{\rho_c - 1} = 1 \tag{90}$$

and

$$\frac{\delta_c}{\Delta_0} = \frac{1}{2\sqrt{\rho_c^2 - 1}} \simeq 0.754. \tag{91}$$

The free energy at the minimum reads

$$\Gamma_{min} = -\frac{4\bar{\mu}^2}{\pi^2}(\rho_c^2 - 1)(\delta - \delta_c)^2 \tag{92}$$

and both branches of eq. (23) are gapless here. The solution satisfies the stability condition (13). Substituting it into eqs.(76), (77), (82) and (83), we obtain

$$A = \frac{g^2\bar{\mu}^2}{96\pi^2}\frac{\hat{\Delta}^4}{(\rho_c^2 - 1)^2} \geq 0, \qquad B = \frac{g^2\bar{\mu}^2}{8\pi^2}\frac{\hat{\Delta}^2}{\rho_c^2 - 1} \geq 0 \tag{93}$$

and

$$C = 0, \qquad D = \frac{g^2\bar{\mu}^2}{6\pi^2}\Big(1 + \frac{e^2}{3g^2}\Big)\frac{\hat{\Delta}^2}{\rho_c^2 - 1} \geq 0 \tag{94}$$

Therefore the chromomagnetic instabilities are removed completely for small gap parameters.

The situation for a multi-plane wave ansatz for the LOFF state will be better if the small gap expansion remains numerically approximate. As we have seen for the single plane wave ansatz, the Meissner tensors are purely longitudinal to Δ^2 and the logitudinal components are positive at the equilibrium. It follows that if the LOFF state (3) contains three linearly independent momenta, the Meissner tensor will be positive definite, since it is additive with respect to different terms of eq. (3) to order Δ^2.

5 Numerical Results

In the last section, we have shown analytically that the chromomagnetic instability is absent in the region near the outer edge of the LOFF window at zero temperature. In this section, we shall explore the interior region in order to determine if the absence of the chromomagnetic instability can be extended to the entire LOFF region at $T = 0$.

Before presenting our numerical results, we shall emphasize some general features of the solutions of eqs. (33) and (34). A trivial solution to eq. (34), $\rho = 0$, corresponds to a homogeneous 2SC and there is always a solution to eq.(33) with $\Delta = \Delta_0$ for $0 < \delta < \Delta_0$. The expression for the free energy then becomes

$$\Gamma = \frac{2\bar{\mu}^2}{\pi^2}\Big(\delta^2 - \frac{1}{2}\Delta_0^2\Big). \tag{95}$$

Since the excitation spectrum, eq. (23), is fully gapped in this case, we shall refer to the correponding state as the BCS state even in the presence of a displacement, following the convention of Ref. [2]. We observe that the BCS state becomes meta-stable relative to the normal phase for

$$\delta > \frac{1}{\sqrt{2}}\Delta_0. \tag{96}$$

For the displacement within the interval $\frac{\Delta_0}{2} < \delta < \Delta_0$, there is another solution at $\rho = 0$ with

$$\Delta = \sqrt{\Delta_0(2\delta - \Delta_0)} \tag{97}$$

which leads to the following expression for the free energy

$$\Gamma = \frac{\bar{\mu}^2}{\pi^2}(2\delta - \Delta_0)^2 > 0. \tag{98}$$

This solution corresponds to the Sarma state that is unstable in the grand canonical ensemble for a given δ. In a canonical ensemble, it may be the only charge neutral state for certain pairing strength [7] but suffers from chromomagnetic instabilities [13].

In what follows we shall restrict the terminology 'LOFF state' to the nontrivial solution to eq. (34) that satisfies

$$\begin{aligned} 1 &- \frac{(\rho+1)^2}{8\rho^3}\Big[3(1-x_1^2)\ln\frac{1+x_1}{1-x_1} + 4(\rho+1)x_1^3 - 6x_1\Big] \\ &- \frac{(\rho-1)^2}{8\rho^3}\Big[-3(1-x_2^2)\ln\frac{1+x_2}{1-x_2} + 4(\rho-1)x_2^3 + 6x_2\Big] = 0 \end{aligned} \tag{99}$$

together with eq. (33). It follows from eq.(33) that

$$\Delta < \Delta_0 \tag{100}$$

at the solution and eq.(99) implies that

$$\frac{\Delta}{\delta} < 1 + \rho. \tag{101}$$

Otherwise, $x_1 = x_2 = 0$ and eq.(99) cannot hold. Physically, eq.(100) is the consequence of the reduced phase space available for pairing and eq.(101) is the condition for the existence of gapless excitations, which is necessary to make the net baryon number current vanish. Combining eq. (100) and (101), we find $\delta < \Delta_0$ for the LOFF state.

For a small gap parameter, an analytical solution was obtained in the last section, which corresponds to a stable state with lower energy compared with the BCS and the normal states and is free of chromomagnetic instabilities. For a general gap parameter, however, eqs. (33) and (99) can only be solved numerically. Our strategy is to find solutions of (99) for Δ/δ for a given ρ and subsequently to calculate the corresponding δ/Δ_0 in terms of (33). Concurrently, we examine the local stability according to eqs. (13) and the competition between the LOFF state we find and the BCS state of the same δ/Δ_0. The Meissner tensors are calculated as well. We observe that the quantities ρ, δ/Δ_0, the free energy difference, the Meissner masses and the stability determinant of eq.

13), at the LOFF solution are single-valued functions of the normalized gap parameter Δ/Δ_0. These functions are plotted in Figs. 1-7.

The diquark momentum of a LOFF state, normalized by the displacement parameter versus the normalized gap parameter is shown in Fig.1 and the corresponding ratio of the displacement parameter to the BCS gap is shown in Fig.2. In Fig.3, we display the free energy difference between a LOFF state and a BCS state at the same δ, normalized by the magnitude of the BCS energy (95) at $\delta = 0$. Combining the three figures, we find the LOFF window, in which the LOFF state is of the lowest energy,

$$\delta_{\rm min} < \delta < \delta_{\rm max}, \tag{102}$$

where $\delta_{\rm max} = \delta_c \simeq 0.754\Delta_0$ and $\delta_{\rm min} \simeq 0.706\Delta_0$, slightly below the threshold (96). The corresponding momentum of the diquark increases monotonically from $\rho \simeq 1.20$ at $\delta_{\rm max}$ to $\rho \simeq 1.28$ at $\delta_{\rm min}$. Our results confirm those of Ref. [2]. In addition, we found that the normalized gap parameter, Δ/Δ_0 varies monotonically from 0 at $\delta_{\rm max}$ to 0.242 at $\delta_{\rm min}$. So the LOFF window on all figures corresponds to the domain $0 < \Delta/\Delta_0 < 0.242$ of the abscissa.

The numerical results of the Meissner tensors, Figs. 4-6 are most interesting. The numbers shown are normalized by the Meissner mass square of the BCS state at $\delta = 0$, i.e.

$$(m^2)_{ij} = A_0\delta_{ij} \tag{103}$$

and

$$(m'^2)_{ij} = D_0\delta_{ij} \tag{104}$$

with

$$A_0 = \frac{g^2\bar\mu^2}{6\pi^2}, \quad D_0 = \frac{g^2\bar\mu^2}{9\pi^2}\Big(1+\frac{e^2}{3g^2}\Big). \tag{105}$$

All squared Meissner tensors are nonnegative within the LOFF window and the region free from the chromomagnetic instabilities extends beyond it. While the Meissner tensor associated to the eighth gluon is aways logitudinal, Fig. 6, the transverse component of the Meissner tensor associated to the 4-7th gluons, Fig.4 is aways much smaller than its logitudinal component, Fig.5. The stability of the solution against virtual displacements of Δ and ρ within the LOFF window is guaranteeted by the positivity of D, plotted in Fig. 6 and the positivity of the determinant in eq.(13), plotted in Fig. 7. The non-smooth behavior of the function D and the determinant of Figs. 6 and 7 correspond to the transition of the LOFF state from the region where both branches of (23) are gapless, $x_2 > 0$, to the region where only the $E_{\vec p}^-$ branch is gapless, $x_2 = 0$. The derivative of x_2 with respect to both Δ and ρ become singular there. When the gap parameter is expressed in terms of ρ and $x_2 > 0$ according to eq. (32) and the function D and the determinant are expanded according to the powers of x_2, the linear term is sensitive to the singularity and gives rise to the non-smooth behavior. On the other hand, the same type of expansion of the functions A and B does not contain a linear term in x_2 and thus the transition behavior of A and B is much more smooth.

As the LOFF momentum goes to zero, $\rho \to 0$, we have $\Delta \to \Delta_0$ and $\delta \to \Delta_0$, a result can be established analytically. Therefore the LOFF state, the BCS state and the Sarma state joins at the point (Δ_0, Δ_0 0) in the three dimensional parameter space of Δ, δ and

ρ. We also observe from Figs. 4-7 that the chromomagnetic instability and the instability against a small variation of parameters show up for sufficiently low values of ρ. Therefore the instability free region of a LOFF state is disconnected from the g2SC state in the parameter space.

6 Concluding Remarks

In this paper, we have systematically examined the Meissner effect in a two-flavor color superconductor with a LOFF pairing. We found that within the LOFF window at $T = 0$, that is within the range of the displacement parameter where the LOFF state is energetically favored, all the squares of the Meissner masses are non-negative. Although the positivity of the longitudinal component of the Meissner mass tensor associated with the eighth gluon follows from the stability of the LOFF state against a variation of the LOFF momentum, ρ, the positivity of the Meissner tensor for the 4-7th gluons appears accidental regarding the pairing ansatz (4). We have also found that the Meissner tensor of the 8th gluon is always longitudinal. This is also the case with the Meissner effect in an electronic LOFF superconductor.

The charge neutrality condition has to be implemented before we can claim that the LOFF state is the ground state of quark matter at moderately high baryon density. In the previous letter [15], we showed that the chromomagnetic instability of the 8th gluon pertaining to g2SC implies a state of lower free energy with a nonzero diquark momentum and without offsetting the charge neutrality. This is not sufficient since the LOFF window, with the diquark momentum comparable to the Fermi momentum displacement, is quite far away from the g2SC and is very narrow in the three dimensional parameter space. It remains to be seen if quark matter within the LOFF window can be made neutral. In a hypothetical world where the sum of the electric charges of the u and d quarks is a small number, we can find within the framework of the weak coupling approximation an analytic expression for an electrically neutral LOFF phase that is located near the upper edge of the LOFF window and is energetically favored over both the neutral normal phase and the neutral BCS phase. Also it is free of chromomagnetic instabilities. The details will be presented in a forthcoming publication [20]. Notice that the neutral phases being compared do not have the same displacement parameter δ since the charge neutrality constraint adjusts δ differently for different phases. In the real world, the sum of the charges of the u and d quarks is 1/3. The weak coupling approximation may be marginal and one may have to seek numerical solutions of eq. (33), eq.(96) and the charge neutrality constraint simultaneously.

We also notice that, if the pairing force is mediated by the long range one-gluon exchange of QCD instead of the point coupling of NJL model, the LOFF window will be at least five times wider under the same Δ_0 ($\delta_{\max} \simeq 0.968\Delta_0$ and $\delta_{\min} \leq 0.706\Delta_0$)[6], making the implementation of the charge neutrality more likely.

In this paper we have considered only the simplest LOFF ansatz that consists of one plane wave and pairs u and d quarks. It was suggested that multi plane wave condensates are favored more near the upper edge of the LOFF window [21]. Numerical analysis also suggests that the s quark cannot be ignored in pairing at $T = 0$. A more complicated

structure of the condensate is required for a realistic LOFF pairing in quark matter and may not be accessible via analytical means. In any case, the absence of chromomagnetic instabilities for the simple 2SC-LOFF is encouraging and promotes the LOFF state as a potential candidate of the ground state of quark matter at moderately baryon densities.

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Appendix A

In this appendix, we shall first derive the exact expression of the quark propagator in the superconducting phase for $\delta' = 0$. Subsequently we shall perform the weak coupling expansion. Finally, we shall approximate the determinant of the quark propagator which enters the free energy function (20) in a similar manner.

As τ_3 and λ_2 are the only isospin and color matrices appearing in the inverse quark propagator (18) at $\delta' = 0$, they can be treated as a c-numbers. Taking the square of the inverse propagator, we have

$$\begin{aligned}[\mathcal{S}^{-1}(P)]^2 &= (i\nu - \delta\tau_3)^2 + \bar{\mu}^2 - p^2 - q^2 - \Delta^2\lambda_2^2 \\ &+ 2\rho_3[\bar{\mu}(i\nu - \delta\tau_3) - \vec{p}\cdot\vec{q}] + 2\Delta\rho_1\gamma_5(-i\vec{\gamma}\cdot\vec{q} + \bar{\mu}\gamma_4)\lambda_2.\end{aligned} \tag{106}$$

In terms of the matrix

$$\begin{aligned}N(P) &= (i\nu - \delta\tau_3)^2 + \bar{\mu}^2 - p^2 - q^2 - \Delta^2\lambda_2^2 \\ &- 2\rho_3[\bar{\mu}(i\nu - \delta\tau_3) - \vec{p}\cdot\vec{q}] - 2\Delta\rho_1\gamma_5(-i\vec{\gamma}\cdot\vec{q} + \bar{\mu}\gamma_4)\lambda_2,\end{aligned} \tag{107}$$

we obtain the exact expression of the quark propagator

$$\mathcal{S}(P) = \frac{\mathcal{S}^{-1}(P)N(P)}{D(P)}, \tag{108}$$

where

$$D(P) = [(i\nu-\delta\tau_3)^2+\bar{\mu}^2-p^2-q^2-\Delta^2\lambda_2^2]^2-4[\bar{\mu}(i\nu-\delta\tau_3)-\vec{p}\cdot\vec{q}]^2+4\Delta^2(\bar{\mu}^2-q^2)\lambda_2^2, \tag{109}$$

and λ_2^2 projects into the subspace of the red and green colors.

In order to perform the weak coupling approximation, we regard $p - \bar{\mu}$, ν, δ and q as small quantities of comparable magnitudes and maintain only the leading order terms in the numerator and the denominator of (108). We find that

$$D(P) \simeq 4\bar{\mu}^2[-(i\nu - \delta\tau_3 - \hat{p}\cdot\vec{q})^2 + (p - \bar{\mu})^2 + \Delta^2\lambda_2^2] \tag{110}$$

and

$$\mathcal{S}^{-1}(P)N(P) \simeq -2\bar{\mu}^2(-i\vec{\gamma}\cdot\hat{p} + \rho_3\gamma_4)[|\vec{p} - \rho_3\vec{q}| - \bar{\mu} + \rho_3(i\nu - \delta\tau_3) + \Delta\lambda_2\gamma_5\gamma_4] \tag{111}$$

Upon substitution of (110) and (111) into (108), we derive the approximate quark propagator expression (21) that appeared in section II.

The determinant of $\mathcal{S}^{-1}(P)$ may be calculated similarly. Let's write

$$\det\mathcal{S}^{-1}(P) = \det^{\frac{1}{2}}[[\mathcal{S}^{-1}(P)]^2] \simeq (2\bar{\mu})^{24}\det^{\frac{1}{2}}d(P), \tag{112}$$

where

$$d(P) = \bar{\mu} - p + \rho_3(i\nu - \delta\tau_3 - \hat{p}\cdot\vec{q}) + \Delta\lambda_2\rho_1\gamma_5\gamma_4. \tag{113}$$

Now by decomposing the 48×48 matrix $d(P)$, according to the eigenvalues of τ_3 and λ_2, i.e.

$$\det d(P) = \prod \det d(P) \mid_{\tau_3=\pm1,\lambda_2=\pm1,0} \tag{114}$$

we have

$$\det d(P) \mid_{\tau_3=\pm1,\lambda_2=1} = [(p-\bar{\mu})^2 + \Delta^2 - (i\nu \mp \delta - \hat{p}\cdot\vec{q})^2]^4, \tag{115}$$

$$\det d(P) \mid_{\tau_3=\pm1,\lambda_2=-1} = [(p-\bar{\mu})^2 + \Delta^2 - (i\nu \mp \delta - \hat{p}\cdot\vec{q})^2]^4 \tag{116}$$

and

$$\det d(P) \mid_{\tau_3=\pm1,\lambda_2=0} = [(p-\bar{\mu})^2 - (i\nu \mp \delta - \hat{p}\cdot\vec{q})^2]^4. \tag{117}$$

Eq. (22) is then established.

Appendix B

The integrand of the angular integrals for the free energy function (28) and that for the Meissner tensor, eqs. (66) and (67), contain terms with $\theta(|\delta_\theta|-\Delta)$. It confines the integral within the region in the phase space where either or both branches of the excitation spectrum, $E_{\vec{p}}^{\pm}$ of (23), become gapless. This occurs when $\Delta < (1+\rho)\delta$. Otherwise the integration domain specified by $\theta(|\delta_\theta|-\Delta)$ is null. There are three different cases to be considered:

1) $|1-\rho|\delta < \Delta < (1+\rho)\delta$:

Only the $E_{\vec{p}}^{-}$ branch may be gapless and we have

$$\int_{-1}^{1} d\cos\theta\theta(|\delta_\theta|-\Delta)(...) = \int_{-1}^{\frac{\delta-\Delta}{\rho\delta}} d\cos\theta(...). \tag{118}$$

2) $\rho < 1$ and $\Delta < (1-\rho)\delta$:

Again we may have only one gapless branch, $E_{\vec{p}}^{-}$, but instead

$$\int_{-1}^{1} d\cos\theta\theta(|\delta_\theta|-\Delta)(...) = \int_{-1}^{1} d\cos\theta(...). \tag{119}$$

because $\frac{\delta-\Delta}{\rho\delta} > 1$.

3) $\rho > 1$ and $(\rho-1)\delta < \Delta < (\rho+1)\delta$:

Both branches, $E_{\vec{p}}^{\pm}$ may be gapless and we have

$$\int_{-1}^{1} d\cos\theta\theta(|\delta_\theta|-\Delta)(...) = \int_{-1}^{\frac{\delta-\Delta}{\rho\delta}} d\cos\theta(...) + \int_{\frac{\delta+\Delta}{\rho\delta}}^{1} d\cos\theta(...). \tag{120}$$

The indefinite integrals of the integrands of the free energy function and the Meissner tensors are all elementary functions.

Appendix C

In this appendix, we shall supply the details supporting the statement in section III that the identity (53) for the quark propagator follows from the invariance of the 2SC-LOFF under a combined transformation of a space inversion, a time reversal and a γ_5 multiplication. We shall work with the canonical formulation at $T = 0$ though the generalization to $T \neq 0$ is straightforward.

The Hamiltonian operator corresponding to the mean field NJL action (9) reads

$$\begin{aligned} H_{MF} &= \Omega\frac{\Delta^2}{4G_D} + \int d^3\vec{r}[\bar{\psi}\vec{\gamma}\cdot\vec{\nabla}\psi - \bar{\psi}\gamma_4\mu\psi \\ &- \Delta(-\bar{\psi}_C\gamma_5\lambda_2\tau_2\psi e^{-2i\vec{q}\vec{r}} + \bar{\psi}\gamma_5\lambda_2\tau_2\psi_C e^{2i\vec{q}\vec{r}})]. \end{aligned} \tag{121}$$

Introduce an anti-unitary operator $\mathcal{U}$ which implements a space inversion, a time reversal and a γ_5 multiplication. The transformation laws of the field operators in Heisenberg representation are given by

$$\begin{aligned} \mathcal{U}\psi(\vec{r},t)\mathcal{U}^{-1} &= i\gamma_2\psi(-\vec{r},-t) \\ \mathcal{U}\psi^\dagger(\vec{r},t)\mathcal{U}^{-1} &= -i\psi^\dagger(-\vec{r},-t)\gamma_2. \end{aligned} \tag{122}$$

The corresponding transformations of the NG spinors are

$$\begin{aligned} \mathcal{U}\Psi(\vec{r},t)\mathcal{U}^{-1} &= i\rho_3\gamma_2\Psi(-\vec{r},-t) \\ \mathcal{U}\Psi^\dagger(\vec{r},t)\mathcal{U}^{-1} &= -i\Psi^\dagger(-\vec{r},-t)\gamma_2\rho_3, \end{aligned} \tag{123}$$

following (5) and (15). The phase factor associated to the time reversal has been fixed in accordance with phase convention of the condensate such that

$$\mathcal{U}H_{MF}\mathcal{U}^{-1} = H_{MF}. \tag{124}$$

For a state $| >$ that is invariant under $\mathcal{U}$, say the ground state and an arbitrary operator O, we have [22]

$$< |O^\dagger| > = < |\mathcal{U}O\mathcal{U}^{-1}| > . \tag{125}$$

The Hamiltonian (121) is also invariant under a unitary transformation, U, that generates a translation and a rephasing according to

$$\begin{aligned} U\psi(\vec{r},t)U^\dagger &= e^{-i\vec{q}\cdot\vec{a}}\psi(\vec{r}+\vec{a},t) \\ U\psi^\dagger(\vec{r},t)U^\dagger &= e^{i\vec{q}\cdot\vec{a}}\psi^\dagger(\vec{r}+\vec{a},t). \end{aligned} \tag{126}$$

Consider the quark propagator in coordinate space,

$$\mathcal{S}_{\alpha\beta}(\vec{r},t) = < |\Psi_\alpha(\vec{r},t)\bar{\Psi}_\beta(0,0)| > \theta(t) - < |\bar{\Psi}_\beta(0,0)\Psi_\alpha(\vec{r},t)| > \theta(-t). \tag{127}$$

It follows from (123), (125), (126) and time translation invariance that

$$\mathcal{S}_{\alpha\beta}(\vec{r},t) = -(\rho_3 C)_{\alpha\gamma}\mathcal{S}_{\sigma\gamma}(\vec{r},t)(C\rho_3)_{\beta\sigma}, \tag{128}$$

which, upon a Fourier transformation, gives rise to the identity (53).

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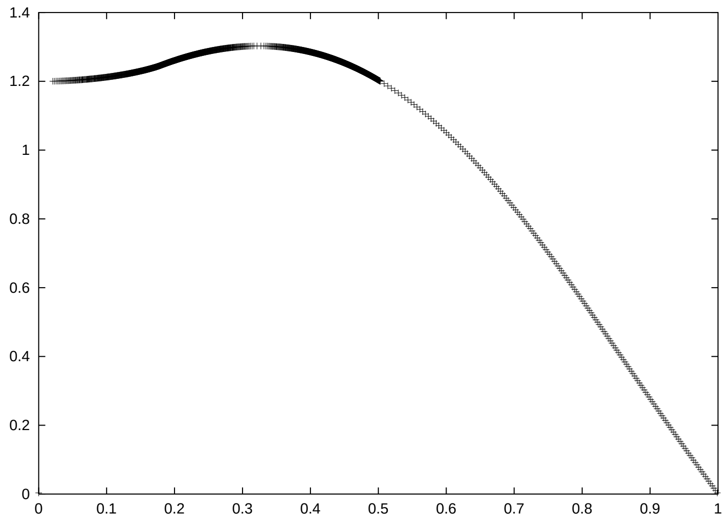

Figure 1: The dimensionless diquark momentum ρ of the LOFF state as a function of the normalized gap parameter Δ/Δ_0.

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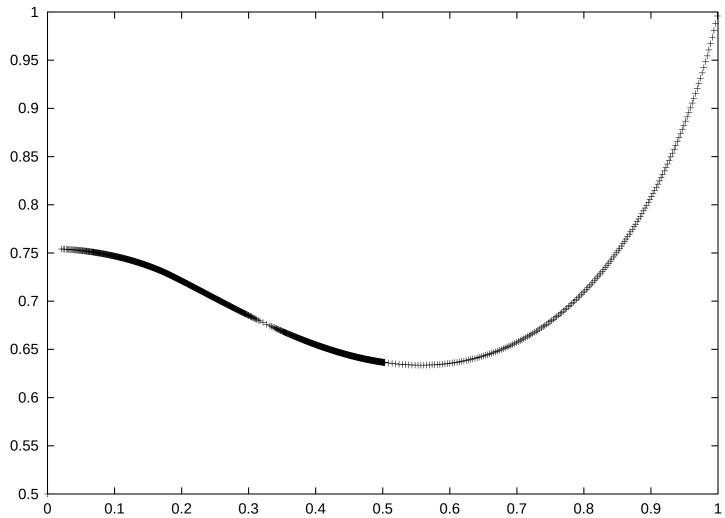

Figure 2: The normalized displacement parameter, δ/Δ_0, of a LOFF state as a function of the normalized gap parameter Δ/Δ_0.

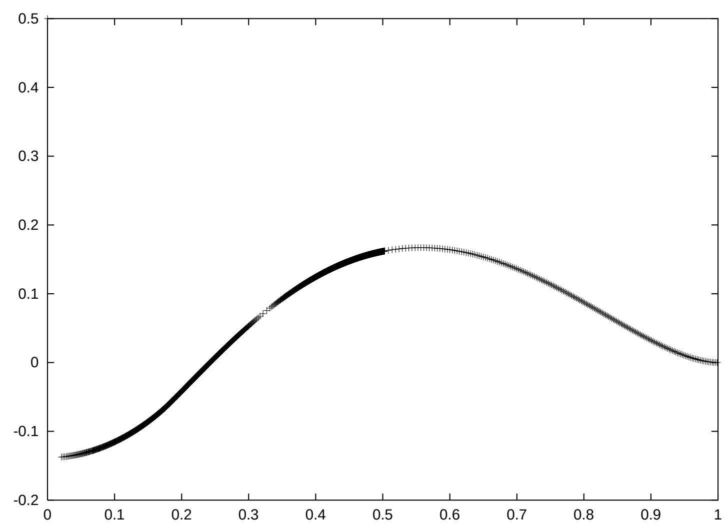

Figure 3: The free energy difference between a LOFF state and a BCS state as a function of the normalized gap parameter Δ/Δ_0. The coordinate is normalized to the magnitude of the free energy of the BCS state, eq.(94), at $\delta = 0$ and the LOFF state is favored when the value is negative.

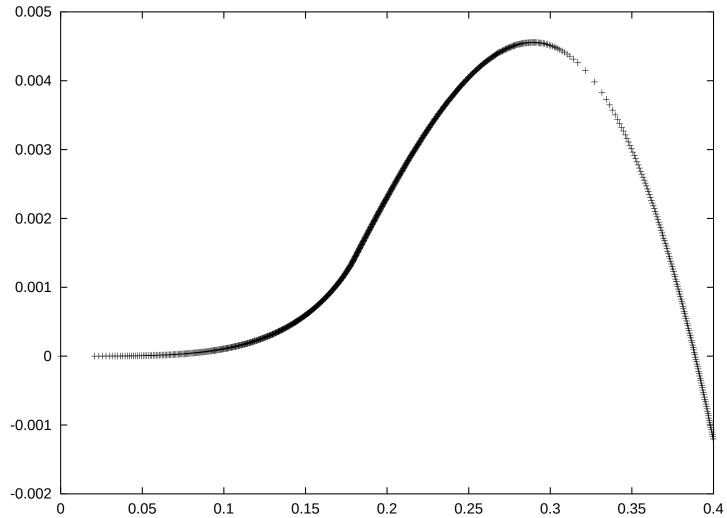

Figure 4: The normalized transverse Meissner mass square of the 4-7 gluons, A/A_0, as a function of the normalized gap parameter Δ/Δ_0 of a LOFF state.

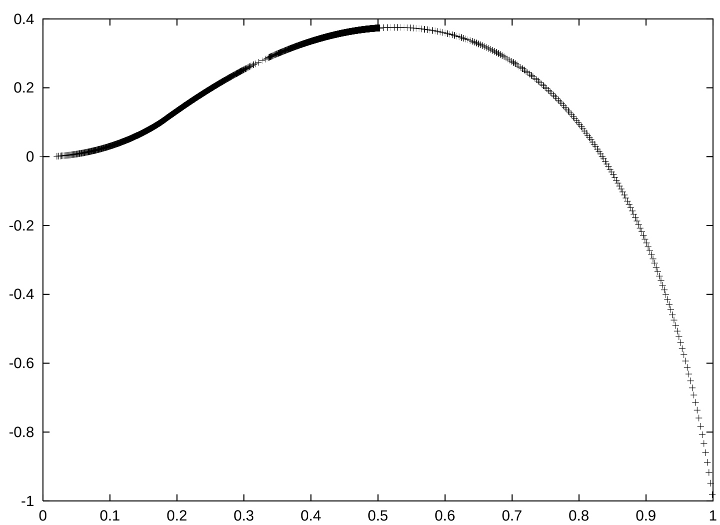

Figure 5: The normalized logitudinal Meissner mass square of the 4-7th gluons, B/A_0, as a function of the normalized gap parameter Δ/Δ_0 of a LOFF state.

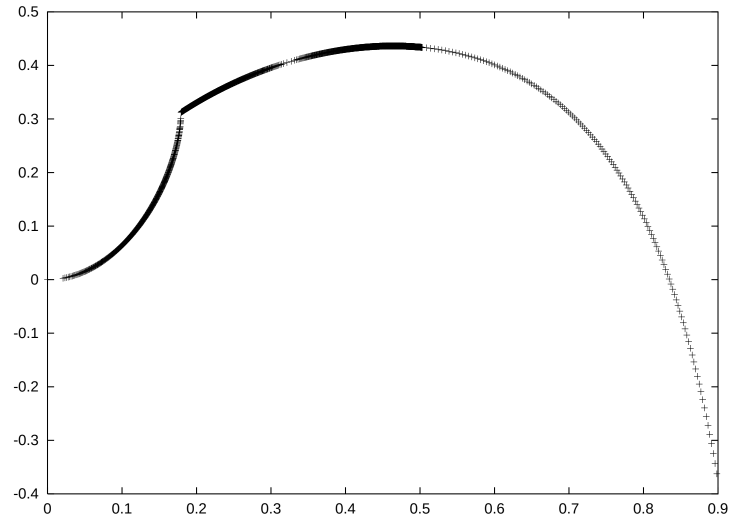

Figure 6: The normalized logitudinal Meissner mass square of the 8th gluon, D/D_0, as a function of the normalized gap parameter Δ/Δ_0 of a LOFF state.

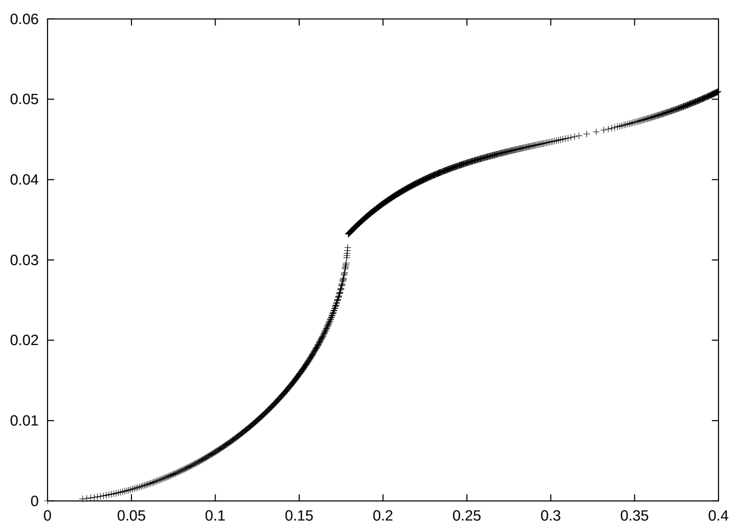

Figure 7: The stability determinant in eq.(13)(arbitrary unit) of a LOFF state as a function of the normalized gap parameter Δ/Δ_0.